Case Study: Effectiveness of technology in students understanding of geometry and probability concepts

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Abstract

The purpose of this case study was to determine if using computer software in classroom instruction would help students learn geometric and probability concepts better. Geometer's Sketchpad and Probability Explorer software was used as treatment instructional tools during a two-week summer Geometry and Probability course. Two groups of students were taught using computer software to determine if the software influenced student learning. The research method used a pre-test/post-test growth measurement indicator. After the completion of two weeks of instruction, students were given a post-test to determine if differences in their achievement was observed. An analysis of the post-test scores using quantitative and qualitative methods indicated that students were better motivated by using computer software. However, results of the data analysis indicated that there was no significant growth in geometric or probabilistic performance by students.

Keywords: Geometry, Probability, Technology, mathematics, student learning, middle school

1.1 Introduction

No Child Left Behind (NCLB, 2001) legislation made educators accountable for students' proficiency in mathematics based on each state's standards of performance. As a result of this, innovative ways have been devised to help students learn mathematics and probability concepts better. In many cases, instructional approaches have turned to technology to help in this enormous task. The National Council of Teachers of Mathematics (NCTM) *Principles to Action* (2014) has endorsed the use of technology for teaching a variety of mathematical content, especially in teaching the concepts of geometry and probability by use of calculators and computers. Using these types of technologies offers the students visual images of mathematical ideas and they facilitate organizing and analyzing data.

Technology in mathematics education aids students in the learning process (NCTM, 2014). NCTM has recognized technology as an essential aspect of the teaching/learning process and has identified technology as one of its principles for school mathematics. NCTM says that "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p.25). Unique technology tools exist to benefit much of what is taught in schools. Examples of technology that exhibit this are computer software such as Geometer's Sketchpad, GeoGebra, Cabri, Fathom, and Probability Explorer.

Geometry learning is vital because it is a fundamental component of mathematical learning. It "offers ways to interpret and reflect on our physical environment and can serve as tools for the study of other topics in mathematics and science" (NCTM, 2000, p. 41). Geometry gives students the opportunity to develop their natural sense of reasoning and justification that are used to build skills across the grades (NCTM, 2000). In its *Principles to Action (*2014) calls for geometry to be learned using a dynamic software as an instructional tool. It is crucial that a new approach to learning these concepts be found because the United States (US) performed poorly in geometry and measurement in comparison to other countries according to the Trend in International Mathematics and Science Study (TIMSS) conducted in 2005 by the American Institute for Research.

Learning probability is also essential because it allows students to be an informed citizen, employee, and consumer (NCTM, 2002). NCTM endorses the learning of probability and feels that students should be taught a full set of necessary skills that include being able to: formulate questions that can be answered with data, analyze data, develop and evaluate inferences, and understand and apply the basic concepts of probability. NCTM further strengthens their position on probability instruction by saying "work in data analysis and probability offers a natural way for students to connect mathematics with other school subjects and with everyday experiences" (p.48). Skills developed in the study of probability will help students in other realms of study, especially in areas of observation and experimental studies. Data from TIMSS & PIRLS (2016) states that 77% of 8^{th} grade students were taught geometric concepts and 60% were taught data and probability. Many software programs exist that are designed to help students' learning in a variety of subjects such as Cabri (Laborde and Bellemain, 2005), The Geometer's Sketchpad (Jackiw, 2001), Fathom (Key Press 2001), and Probability Explorer (Lee, 2001).

The purpose of this study was to determine the effectiveness of dynamic computer software on the learning of geometry and probability concepts. Moreover, the study analyzed student outcomes on pre and post-tests on geometry transformations and concepts of probability with replacement.

2.1 Dynamic Geometry

Learning geometry can be complicated for students, because of the abstractness of the concepts. An example of this would be students' understanding of transformations. Students often try to understand transformations by doing them in their heads or with static representations given by pencil and paper. In many cases, students fail to make accurate constructions of the figures on which they are trying to perform transformations (Hollebrands 2003). Although students use protractors, rulers, and compasses to create transformations, over time students become wary of the tasks because of the amount of time and steps involved to make accurate figures. Typically, students mimic their instructors and go through exercises without learning and developing needed skills and understandings (Chazan, 1993).

The use of geometry technologies can be incorporated into instruction as a tool for both students and teachers. There have been many studies that incorporate dynamic geometry software into classroom instruction (Chazan, 1993; Edwards, 1997; Hollebrands, 2007; Hoyes & Noss, 1994; Laborde, 1993; Mariotti, 2000; Sanchez, Ernesto, & Sacristan, 2003). Dynamic geometry software gives users the ability to create constructions using similar techniques of those associated with a straightedge and compass as well as techniques and methods unique to this dynamic environment. The use of dynamic software enhances the opportunity for students to learn geometric properties associated with constructions they perform because such constructions may be easily repeated using different figures. The added function of being able to drag computer-generated objects also benefits the learning process. Previous research in dynamic geometry software use has produced insight in two areas of importance: "(a) the effects on students' understandings of geometrical figures and properties, and (b) its influences on students' deductive reasoning" (Hollebrands & Smith, 2009). In this study, we are concentrating on the effects on students' understanding of geometrical figures and properties in contrast to using traditional classroom methods to teach these concepts.

Dynamic geometry software empowers students to perform a variety of tasks and encourages them to develop geometric conjectures about observed relationships. This instructional tool is beneficial to students because it strengths their prior knowledge of a protractor, ruler, compass, and enables them to create and recreate figures quickly. Students can do more examples of similar problems with less frustration and more accuracy. Students are more likely to understand why deductive proofs are needed, in contrast to empirical proofs, and understand the meaning of geometric properties. For example, students can quickly measure angles and length accurately through guided discovery exercises designed to help them develop conjectures about geometric properties. Students can investigate empirically and understand the deductive proof without being able to create the proof from the beginning (Chazan, 1993).

The research in using dynamic geometric software to explore geometric transformations brings two critical aspects to the fore regarding the understanding of geometry in a dynamic software environment. The distinction between drawing and constructing a figure and how computer environments mediate students' understanding of geometry are important notions (Hollebrands, 2007 p.168). Hollebrands (2007) analyzes these in her study by giving examples of drawings versus constructions and what makes them different.

From this research, it was found that it was possible for a teacher and student to interact within a dynamic geometry environment and both adopt and use a common language under mathematical ideas represented by shared visual images. These results are similar to findings discovered by Vygotsky (1978), in which he states that "it is decisively important that speech not only facilitates the child's effective manipulation of objects but also controls the child's behavior" (p.26).

Verbalizing their thoughts can be quite a challenge for students. However, while students are being challenged, they develop problem-solving, critical thinking, and analytical skills. Engaging students in activities and games involving geometry is a powerful way to help students develop essential understandings and skills. When so engaged, students often do not realize that they are learning and putting together geometric properties that they have been studying (Edwards, 1997).

In the case of a reflection, the student would be able to see that once the figure was moved, the reflection of the figure was in one-to-one correspondence to the original figure. Students can see hundreds of examples like this. According to Hollebrands (2007), technology makes the actions of transformations visible to students in a dynamic environment in such a way that students develop deeper understandings of transformations.

In a study by Edwards (1997), students learned how to use the TGEO microworld to perform geometric transformations. The microworld allowed students to superimpose one shape on another. The activity became a game to the students who could only use geometric transformations to succeed at the task. The "game" helped students develop a working understanding of transformations and provided an environment in which to explore transformations through guided discovery. Although upon completion of the study students were not able to develop their proofs, they were able to follow and understand proofs provided by the investigator.

Geometry software played an essential role in the studies of Chazan and Edwards. The studies were able to establish that computers could be used by students to learn critical geometric concepts and properties. In both studies, students were able to test and explore conjectures made in a more manageable environment. Students were then able to transfer their understanding to traditional situations in their study of geometry.

2.2 Dynamic Probability

Learning and teaching probability concepts in middle grades can be a difficult task for many teachers. They may find that when they first begin to teach probability concepts that they are not sure where to start in the development of the necessary constructs. Researchers (Borovcnik, Bentz, & Kapadia, 1991; Hawkins & Kapadia, 1984; Shaughnessy, 1992) suggest that teachers should start with an experimental approach that relies heavily on objective analysis of repeated experiments. The traditional approach found in most curricula begins with mathematical axioms, rules, and numerical activities that promote memorization rather than understanding. The axioms usually start with pure probability and scaffold to more advanced axioms such as conditional probability. Students often do not grasp the basic concepts from mathematical axioms and rules because students lack the cognitive development needed. It is inferred that students do not ask questions of "how" and "why" because they see probabilistic thinking as a skill and not a process of critical thinking.

There exist many different arguments for how probability should be taught (Borovcnik & Bentz, 1991; Streinbring, 1991; and Shaughnessy, 1992). They vary from traditional rule-driven theoretical approaches to skilled-oriented instruction. Neither of which has been categorized as being useful in teaching students how to effectively use probabilistic reasoning in new situations (Stohl, 2001). Although experimental approaches are often used in school curricula to teach probability concepts, very few employ technological tools.

Dynamic software exists for the sole purpose of helping students understand probability concepts. They include Tabletop Jr., Graphers, Probability Toolkit, Probability Constructor, and Probability Explorer. Although these software programs are commercially available, limited studies (Stohl, 2001) have been conducted to see how effective they are in helping students develop probabilistic reasoning. Konold (1991) developed and used ProbSim software with his students and found that students were able to use the simulations that it created to analyze data more effectively. Like Konold, others developed simulation software to meet the need for analyzing empirical data. ChanceWorld, developed by Jiang and Potter (1994), was used in a study in which it was found that the use of the computer environment was beneficial to students in helping them overcome common misconceptions about probability. The software program being used in this study is Probability Explorer (PE).

The Probability Explorer software was developed and used by Stohl (2001) for her dissertation work on probabilistic reasoning in children using microworlds. "The design of the microworld is based on a constructivist theory of learning, the design of mathematical computer microworlds, and research on students' understanding of probability and rational number concepts" (Drier, 2000). Implications from Stohl's work suggest that carefully designed microworld tools can facilitate probabilistic reasoning. From her work, she was also able to conclude that reflecting on changes in multiple representations during a simulation encouraged students to develop theories-in-action about the Law of Large Numbers (Stohl, 2001).

3.1 Methods

The purpose of the study was to determine if using dynamic computer software in classroom instruction would help students learn geometric and probability concepts. The instructional variables in the study were traditional classroom instructional methods and instruction involving a dynamic computer software. The students were interviewed to obtain background and demographic information. Student learning was measured by their performance on the pre- and post-tests that were given at the beginning and end of the investigation period.

3.2 Participants

Nine (9) middle grades students from a public-school system in central North Carolina participated in this research study. The students ranged in age from eleven to thirteen years old. Seven (7) male students and two (2) female students took part. The two female students were both African American. Six male students were African American, and one was Asian American. All the participants were average or above average students (grade C or better) in mathematics. All the students had necessary computer skills in that they knew how to use the mouse and keyboard; they knew how to open computer software programs, and how to follow software program instructions. Student interviews revealed that they had not been exposed to very much geometry or probability instruction in their formal schooling to date. The socio-economic status of all the students was determined to be middle class. None of the students had any documented disabilities—physical, emotional, or mental.

The two instructors (Instructors A and B) who taught the geometry and probability classes of the study had stable backgrounds in mathematics. Both were experienced middle school teachers with a combined seven years of public school classroom teaching at the middle school level.

3.3 Treatment

Students in the study were divided into two groups: Group 1 and Group 2. Instructor A taught both geometry and probability using the appropriate dynamic software programs (Geometer's Sketchpad and Probability Explorer). Instructor B taught both geometry and probability in a traditional manner without using any dynamic software programs.

The study was conducted over a two-week period. Each group of students received two-hour-long lessons each day of the study. See Table 3.1 for the study's research design.

Table 3.1 Research Design

Research Design		
	Group 1	Group 2
Geometry (Week 1)	Sketchpad-based Instruction (Instructor A)	Traditional Instruction (Instructor B "control")
Probability (Week 2)	Traditional Instruction (Instructor B "control")	Probability Explorer-based instruction (Instructor A)

Pre- and post-test scores were analyzed. The research hypotheses were as follows:
H1: No significant differences exist between the pre- and post-test geometry scores of Group 1 & Group 2. (P-value ≤ 0.05)
H2: No significant differences exist between the pre- and post-test probability scores of Group 1 & Group 2 (P-value ≤ 0.05)

A group of nine (9) sixth-grade students were randomly divided into two smaller groups. The first group (Group 1) had five students and the second group (Group 2) had four students. Each participant was interviewed by the researcher who asked seven questions about their experiences with technology and their likes and dislikes regarding the use of technology to learn mathematics. The interviews helped to gain more information about students' feelings toward the math content. These data were recorded and saved for analysis and comparison to post-interview data.

A pre-test and a post-test were created for the geometry and probability treatments so that students' scores could be compared at the end of the study. The geometry and probability pre-post tests were created from the teaching materials that the instructors used in the class. Problems from each day of instruction were selected from in-class materials. The problems covered all the instruction that took place over the period of the study. At the beginning of the study, the two groups of students were given two pre-tests to measure their understanding of geometry and probability. At the end of the first week, a post-test in geometry was administered to Groups 1 and 2. At the conclusion of the second week, a post-test in probability was administered to Groups 1 and 2.

3.5 Statistics

A two-sample t-test was used to detect differences between the groups' performance on the pre- and post-tests. The t-test allows us to observe differences between two samples with the results depending on the spread of the observations as well as on the two means (Moore, 2004). It was assumed that the variances were unequal in all the cases.

3.6 Materials

The classroom in which the traditional instructional approach was implemented was a typical mathematics classroom. The classroom in which the instructional treatment was implemented was equipped with a presentation device and a projection screen on which the teacher could project software-driven instruction. Students receiving the instructional treatments used handouts provided by the instructor. Each student in the treatment groups had access to a Dell Latitude laptop computer with wireless internet capability and a Texas Instruments 83-Plus calculator. The computers were equipped with Geometer's Sketchpad and Probability Explorer software packages. Students also used a Dell flash drive to save their work from the laptops.

3.7 Instruction

Instruction for the Geometer's Sketchpad treatment was taught using supplemental handouts developed by Karen Hollebrands (2006). Hollebrands (2006) created these materials, which were modified in the study for middle school students, to teach high school students geometry by incorporating dynamic geometry software instruction into their geometry curriculum. Instructor A was a facilitator for his students, providing help and support as needed. Each day's lesson included a discussion about the lesson objectives designed to help students better understand the content. Geometry: Applying, Reasoning, Measuring (Larson, Boswell, & Stiff, 2000), pages 394-412, was used as a reference for definitions and study problems. The traditional geometry class used the same textbook as its source of instruction. Standard tools such as protractors, pattern blocks, wax paper, and the chalkboard were used in the traditional classroom to help the student to learn lesson objectives.

Instruction for the Probability Explorer treatment was taught using supplemental handouts and computer-generated files developed by Hollylynne S. Lee (2006). No textbook was used in this class; students were taught probability using the handouts provide. The traditional probability class used number cubes, spinners, playing cards, and the chalkboard to help the student to learn lesson objectives.

On the first days of geometry and probability instruction, both Groups 1 and two were given a pre-test to determine their existing knowledge and skills for the subject matter. The participants were taught geometry during the first week. Group 1 was randomly determined to receive the treatment instruction using the Geometer's Sketchpad software. During the first week, Group 2 received traditional geometry instruction. Consequently, Group 2 would receive the Probability Explorer treatment for the probability instruction given the second week. Accordingly, Group 1 would be taught probability using traditional teaching methods.

3.8 Geometry Treatment

During Week 1 of the geometry lessons, both groups studied transformations: reflections, rotations, and translations. On Day 1, the students took a pre-test on transformations and were introduced to transformations.

The treatment group, Group 1, received its introduction to the software Geometer's Sketchpad and received handouts on the names of transformations: "Explorations I-V" (Hollebrands, 2006). Instructor A used Geometer's Sketchpad to complete the handouts. The control group, Group 2, used traditional teaching methods to meet the same lesson objectives. On Day 2, a review of the previous day's lesson was provided to both Groups 1 and 2. Then, the new lesson objectives were introduced. The treatment group, Group 1 was taught translations using Geometer's Sketchpad. The students were given handouts to complete the tasks in translations. The control, Group 2, was taught using materials such as protractors to demonstrate translations. This pattern of instruction for the treatment and control group follows throughout the rest of this research project. On Day 3 of the project, a review of translations was given to both groups. Then, appropriate lessons on rotations were introduced to each group. On Day 4 a review of rotations was provided, and new lessons on reflections were taught. On Day 5, a review of all the transformations was provided followed by a post-test.

3.9 Probability Treatment

The lesson objective taught in Week 2 of the project was about how to compute simple probability with replacement. On Day 1, students took a pre-test on probability and were then introduced to computing simple probability with replacement. Instructor A taught Group 2 students how to use Probability Explorer. They were then shown how to compute simple probability with replacement using handouts that were provided. The Probability Explorer software helped students in computing simple probabilities by demonstrating the connections between theory and empirical situations. Probability Explorer did not compute the probabilities for students but became a visual aid in helping them see what was going on and keeping track of data. Group 1, now the control group, learned simple probability from using manipulatives, number cubes, and handouts. This pattern of instruction for the treatment and control groups was followed throughout the rest of the week. That is, prepared activities that were used to teach the probability concepts were used with both groups, but Group 2 used Probability Explorer to explore relationships while Group 1 used hand-held manipulatives. On Day 2, a lesson on simple probability with replacement was continued. The "Guess My Marbles I, Guess My Marbles II and Guess My Marbles III" activities were designed to teach simple probability with replacement (Lee, 2006). These activities allowed Group 2 students to create their experiments using Probability Explorer as an instructional tool. The control group, Group 1, used manipulatives to achieve a similar experience. On Day 3 of the project, the simple probability with replacement was continued using "The Fish Study" (Lee, 2006). The fourth and final day of the teaching experiment, students reviewed the lesson objective that was covered during the four-day period.

4.1 Pre-Interviews

The students who participated in the study were interviewed before their instruction to acquire information about their backgrounds and academic status. On the pre-interview, 5 out of 9 students said they enjoyed learning mathematics. All the students said they enjoyed using technology. Each child said they had a computer available to them at home. Although the students in the study indicated that they had been exposed to using technology in an educational setting, only 2 out of 9 of them had used computer software to do either geometric or probability assignments. All the students did indicate that they felt that using computer software would help them to learn geometric and probability concepts in a meaningful way.

4.2 Post-Interviews

All the participants felt that the technologies that they were exposed to helped them learn the geometric and probability concepts they were taught in the study. The students expressed how much they enjoyed using the geometry and probability software and completing the activities during instruction. When asked if they now enjoyed mathematics after being exposed to the math concepts in a new way, 8 out of 9 students stated that they now enjoyed mathematics after using the computer software to learn mathematical ideas. We observed an increase of 3 students, which now enjoyed math because of the technology. This could indicate that technology can be used to motivate students in learning new mathematical concepts.

4.3 Data Analysis

Table 4.1 shows the scores and the differences between scores on the pre- and post-tests given in geometry and probability.

Table 4.1 Pre-Post Test Scores

Geometry

Group (1 or 2)	Treatment Type & Instructor (A or B)	Pre-test	Post-test	Difference
1	Sketchpad(A)	35	35	0
1	Sketchpad(A)	35	65	30
1	Sketchpad(A)	25	60	35
1	Sketchpad(A)	10	5	-5
*1	Sketchpad(A)	50	45	-5
	average	31	42	11
2	Traditional(B)	25	40	15
2	Traditional(B)	55	85	30
2	Traditional(B)	30	85	55
2	Traditional(B)	65	55	-10
	average	43.75	66.25	22.5

Probability

Group (1 or 2)	Treatment Type & Instructor (A or B)	Pre-test	Post-test	Difference
1	Traditional(B)	29	33	4
1	Traditional(B)	14	29	15
1	Traditional(B)	14	33	19
1	Traditional(B)	29	29	0
	average	21.5	31	9.5
2	Probability Explorer(A)	38	33	-5
2	Probability Explorer(A)	29	38	9
2	Probability Explorer(A)	24	29	5
2	Probability Explorer(A)	24	33	9
	average	28.75	33.25	4.5

*Note “one traditional student” dropped out of second study on Probability

Tables 4.2 and 4.3 show mean difference scores and standard deviations for the pre-test/post-test differences for geometry and probability, respectively. It was found that the control group (Group 2) in geometry had a higher mean difference score than the treatment group (Group 1). Difference scores were analyzed using Student’s t-test for $\alpha = .05$. The results of this analysis are found in Table 4.2. As seen in the table, there was no significant difference between the mean difference scores in geometry for Groups 1 and 2.

Table 4.2 Descriptive Statistics for Geometry

t-Test: Two-Sample Assuming Equal Variances
Pre-Test / Post-Test Difference for Geometry

	Treatment Geometry	*Traditional (control)*
Mean	**11**	**22.5**
Variance	392.5	741.6666667
Observations	5	4
Hypothesized Mean Difference	0	
df	5	
t Stat	-0.707887807	
P(T<=t) one-tail	**0.255319356**	
t Critical one-tail	2.015048372	
P(T<=t) two-tail	0.510638711	
t Critical two-tail	2.570581835	
Standard Deviation (Treatment Geometry)	**19.81161276**	
Standard Deviation (Traditional Geometry)	**27.23355773**	

Similarly, it was found that the control group (Group 1) in probability had a higher mean difference score than the treatment group (Group 2). Again, difference scores were analyzed using Student's t-test for $\alpha = .05$. The results of this analysis are found in Table 4.3. As seen in the table, there was no significant difference between the mean difference scores in probability for Groups 1 and 2.

Table 4.3 Descriptive Statistics for Probability

t-Test: Two-Sample Assuming Equal Variances
Pre-test / Post-test Difference for Probability

	Treatment Probability	*Traditional (control)*
Mean	**4.5**	**9.5**
Variance	43.66666667	80.33333333
Observations	4	4
Hypothesized Mean Difference	0	
Df	6	
t Stat	-0.89802651	
P(T<=t) one-tail	**0.201882952**	
t Critical one-tail	1.943180274	
P(T<=t) two-tail	0.403765903	
t Critical two-tail	2.446911846	
Standard Deviation (Treatment Probability Explorer)	**6.608075867**	
Standard Deviation (Traditional Probability)	**8.96288644**	

5.1 Discussion

According to Table 4.2, students did not benefit from using Geometer's Sketchpad to learn concepts about geometric transformations. There is an 11.5-point difference in the averages suggesting that traditional classroom methods used to teach transformations to work better than dynamic geometry software. However, if we look back at Table 4.1, you can see that one student's improvement of 55 points could easily skew the overall mean.

In geometry, the post-test scores of students who had Instructor B were, on average, 24 points higher than Instructor A. In probability, the post-test scores of students who had Instructor A were approximate, 2 points higher than Instruction B. (See Table 4.1.) From the data, we can say that Instructor B was better at teaching the probability and geometry lesson objectives as evident by the mean difference scores in Tables 4.2 and 4.3.

On the other hand, we note that the students in Instructor A's classes had to learn more in the same period according to Table 4.4. The students had to learn how to use the computer software, thus giving them less time to concentrate on the learning objectives. Undoubtedly, this represents a limitation of the study.

Table 4.4 Topics of Instruction

Topics covered	
Instructor A	***Instructor B***
Learn software basics	Teach specific concepts
Teach specific concepts	Ensure students are using traditional manipulates correctly
Teach software basics for that specific concept (repeated for each new concept)	Guided Practice Time for Students with Instructor
Ensure students using software correctly	
Guided Practice Time for Students with Instructor	

The t-test was used to analyze the data for the geometry pre-test/post-test difference scores. The hypothesis was that the geometry software treatment scores would be the same as traditional treatment scores with p-value equal to .05. The t-test yielded a one-tailed p-value of 0.25. Therefore, we conclude that no significant difference exists between the geometry software treatment scores and traditional treatment scores.

The t-test was also used to analyze the data for the probability difference scores. The t-test yielded a one-tailed, p-value of 0.20. Therefore, we conclude that no significant difference exists between the probability software treatment scores and traditional treatment scores.

5.2 Conclusions

The purpose of this study was to find evidence that suggests that using computer software would make a difference in students learning geometry and probability concepts. The research study was conducted by having students separated into two groups with one group as the traditional (control) and the other as the treatment group. The students took pre and post-tests on the content of the study. The scores from these tests were analyzed using quantitative methods.

The statistics from the geometry part of the study suggested that students in the traditional classroom environment outperformed those in the computer software-oriented classroom. Although the use of software did not result in students performing better than their counterparts, it is believed, based on post-interviews that the computer-oriented classrooms did have an impact on how the students felt about the subject matter. The students were more motivated to complete the task because they were interested in using the computer to do so. Still, it can be noted that there was no significant difference between the software treatment group and the traditional group.

The second part of the research study looked at probability concepts. The post-test scores do not have evidence that suggests students scored better when they were able to use the probability software. The statistical analysis suggests that there is not a significant difference in the post-test in either case. It cannot be concluded that students who learned probability using the software did better than students who did not. Students who used the software did not do better we feel because they had to learn how to use the software in a small window of time. The software allowed the students to build tables of compiled data, which seemed to enhance student understanding of related probability concepts, but the data did not reflect that.

By constructing examples, the students were able to explore learning probability in different ways that could be more meaningful to them. It was also noted that a teacher effect might exist. However, it was beyond the scope of this study.

The data from this study suggest that a teacher effect may be present. Instructor B had better overall results than Instructor A. Any future study must account for instructional differences according to the teachers participating.

Lastly, it appears that using computer software to help students learn geometric and probability concepts are used as a motivational tool as conclude from the post-interviews. We found that students enjoy using computer programs during instruction and there is evidence that students are actively engaged because of using computer software to complete classroom tasks. Unexpectedly, we found no evidence to infer that software used in geometry and probability helps students to learn geometry and probability concepts better or faster than by traditional methods.

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