Considerations for Teaching First Semester Calculus

Joel R. Helms
Associate Professor - Mathematics
Brigham Young University-Hawaii
United States of America

Abstract

As a consequence of weak algebra skills, many first semester calculus students have difficulty understanding worked examples when instructors skip algebraic steps when teaching. Cognitive load theory reveals why skipping steps hinders learning when students lack algebra proficiency. Cognitive load theory also reveals why a traditional approach to teaching calculus is effective when written classroom examples are well-organized and include verbal explanations.

Research literature indicates, as students' abilities change instructional practices should change as well. Because students are exceptionally weak in algebra at the beginning of calculus, instructors should avoid skipping algebraic steps at the beginning of the course. As students' algebra proficiency steadily increases, instructors should gradually skip algebraic steps when writing worked examples in class.

Keywords: Chunking, Cognitive Load Theory, high-element interactivity, low-element interactivity, schema.

Introduction

Background of the Problem

Weak Algebra Skills

Many students entering college are weak in algebra including students majoring in science, mathematics, and engineering (Bailey, Jeong & Cho, 2010; Budny, Bjedov & LeBold, 1998; Jourdan, Cretchley & Passmore, 2007). Between 1966 and 1993, engineering students at Purdue University dropped engineering as their major due to struggles in calculus (Budny, Bjedov & LeBold, 1998). Science departments at Purdue performed studies to determine why so many Purdue students struggle in courses such as physics, chemistry and calculus. The findings indicated weak algebra skills were the underlying problem in all the said sciences courses, including calculus (Budny et al., 1998).

In a study by Jourdan, Cretchley and Passmore (2007), new college students at an Australian university, whose majors were engineering or science, were found to be weak in algebra. Over 40% of the students were unable to factor basic quadratic expressions and solve simple quadratic equations. Fifty-nine percent of the Australian students were incapable of subtracting two rational expressions and, given $f(x)$, 61% were unable to find $f(x+h)$

In a study by Orton (1983), 110 calculus students from both high schools and colleges were clinically interviewed. Orton found these students struggled with solving basic algebraic equations. In the process of working calculus problems, Orton found several students were unable to correctly solve the quadratic equation $3x^2 - 6x = 0$. Not only did students make procedural mistakes but, many gave one solution, suggesting several students could not identify the equation as quadratic or they did not understand quadratic equations have two solutions. In other words, many students lacked both procedural and conceptual understanding.

In another study by Martin (2000), which focused on large urban university calculus students' ability to solve related rates problems, the author assumed students had considerable prior experience with solving algebraic equations. The author also assumed students were proficient in algebra. Students did in fact have a decent amount of experience but, Martin found students lacked fundamental algebra skills. Martin's conclusion was students' prior achievement in algebra does not imply algebra proficiency nor sufficient readiness for calculus.

Even when students are well prepared in algebra professors of science, math, and engineering (SME) often claim students should be weeded out (Daempfle, 2003). SME faculty also believe the intrinsic difficultly of SME is a key contributor to high attrition rates (Daempfle, 2003). However, such claims are founded on opinion and not from research literature (Seymour & Hewitt, 1994). For example, despite the claim of intrinsic difficulty being a main reason for attrition, only 12% of mathematically prepared SME students change majors because of intrinsic difficulty (Daempfle, 2003). In a three year study with 335 students from seven universities, Seymour and Hewitt (1994) found the primary reasons mathematically prepared students switch their major from SME to another major is because of ineffective teaching and indifferent attitudes of instructors. Whether students are sufficiently prepared mathematically or not, ineffective classroom instruction contributes to high attrition rates in SME (Daempfle, 2003).

Instructional Design

Quality instruction in mathematics courses is positively correlated with student learning (Carrell & West, 2008). Because a large amount of traditional calculus (as opposed to calculus reform) relies on students performing algebraic computations, algebra proficiency is crucial for success (Heid & Edwards, 2001; Tall, 1993). Yet students entering first semester calculus are generally weak in algebra (Budny et al., 1998; Jourdan et al., 2007; Martin, 2000; Orton, 1983). Hence the question, what approach to teaching first semester calculus is effective given that many students are weak in algebra? In this study, Cognitive Load Theory (CLT) will be the basis for developing methods of instruction which lead to proficiency in algebra while learning the calculus.

Statement of the Problem

Skipping Steps

Similar to the assumption made by Martin (2000), many college professors assume first semester calculus students are sufficiently prepared in algebra (Avila, 2013). Nevertheless, research literature indicates many, if not most calculus students are weak in algebra (Budny et al., 1998; Jourdan et al., 2007; Martin, 2000; Orton, 1983). Calculus professors who are cognizant about students' weakness in algebra might address the weakness by spending the first week of class reviewing algebra. A week of algebra review helps to hone algebra skills, however, a week of review does not allow enough practice over an extended period of time to reach proficiency in algebra. To understand how to apply algebra in calculus requires procedural knowledge (Ritter, Anderson, Koedinger & Corbett, 2007). To attain procedural knowledge a substantial amount of time, practice, and effort is needed (Pollock, Chandler & Sweller, 2002; Salden, Paas, & Van Merriënboer, 1994).

Between the first and last day of calculus, students gradually become more competent in algebra. Competence takes time and instructional techniques must be designed around this gradual change (Ritter, Anderson, Koedinger & Corbett, 2007). Therefore, skipping steps should be avoided towards the beginning of the semester but, instructors should gradually skip steps as the semester progresses (Kalyuga, Ayres, Chandler & Sweller, 2003).

Purpose of the Study

Many postsecondary calculus professors assume first semester calculus students are sufficiently prepared in algebra and, those who are not should be weeded out (Avila, 2013; Daempfle, 2003). Most calculus textbooks contain one chapter or less of review and the review is neither algebraically intensive nor comprehensive (Herriott & Dunbar, 2009; Stewart, 2015; Tan, 2011). As a result, students will likely miss important information, make mistakes, and perform poorly (Ambrose, Bridges, DiPietro, Lovett & Norman, 2010). Through cognitive load theory assessment, the purpose of this study is to substantiate an effective approach to teaching first semester calculus given that students enter calculus with weak algebra skills but, gradually become more proficient in algebra as the semester progresses.

Literature Review

Classification Theory

Classification theories are built on the concept of classifying objects according to their properties (Hjørland & Pedersen, 2005). Classifying objects assumes one has some sort of knowledge about the object. If the classifier lacks knowledge or, the knowledge is not explicitly understood, the classification made has a high probability of being incorrect (Hjørland & Pedersen, 2005). If knowledge is lacking the student must acquire the knowledge but, according to CLT, if information is misunderstood the associated schema is poorly formed. Poorly formed schema result in irrelevant or incorrect information entering the working memory (WM) (Bull & Scerif, 2001). On the contrary, if a student can accurately classify an equation the associated schema is well-constructed.

Due to well-configured schema, the expert in algebra can classify an equation as linear, quadratic, exponential, logarithmic, etc. The expert can also identify what procedure should be used to solve the equation (Krathwohl, 2002; Ritter et al., 2007). For instance, assume a given equation is quadratic in form. Because the schema in the expert's LTM is well structured, the expert can quickly classify the form as quadratic (Hjørland & Pedersen, 2005). Students who can accurately classify an equation are more likely to know the procedure to solve the equation (Hiebert, 2013). Explicitly stated, the expert in algebra can accurately classify an equation and identify the procedure required to accurately solve the equation (Krathwohl, 2002; Ritter et al., 2007). However, relative to algebra, most students entering first semester calculus are novice learners and therefore lack the ability to correctly classify equations (Budny et al., 1998; Jourdan et al., 2007; Martin, 2000; Orton, 1983).

Mental Calculations and Skipping Steps

When doing mental calculations, errors happen because the information required while performing the calculations are held in the WM and are prone to forgetting (Cooper, 1998; Hitch, 1978). Adams and Hitch (1997) found how one's ability to perform mental arithmetic is limited by one's proficiency in arithmetic due to the limited storage capacity of the WM. The less proficient one is in arithmetic the lower their capacity to do mental arithmetic because those who are less proficient have yet to chunk concepts in the LTM (Gobet, 2005). Un-chunked concepts are treated as single elements in the WM but, the WM can only handle a few single information elements at one time (Paas & Ayres, 2014; Van Merriënboer & Sweller, 2005). It seems reasonable to conclude the same would hold true for algebra due to the universal nature of chunking (Anderson, 2013; Gobet,2005; Koedinger & Anderson, 1990). Relative to algebra, one's ability to perform mental algebra (skip algebraic steps) is limited by one's proficiency in algebra. Because many calculus students are not proficient in algebra, most calculus students' ability to skip algebraic steps would be limited (Budny et al., 1998; Helms, 2018; Martin, 2000; Orton, 1983).

Hitch (1978) also discovered how fewer mental mistakes were made when the results of each [prior] mental calculation were recorded in written form. When calculations are written, the hardcopy serves as an efficient external working memory which frees up space in the WM for the next mental calculation (Cooper, 1998; Lindsay & Norman, 2013). The findings of Hitch, Cooper, and Lindsay and Norman offer evidence that providing a visual [written] record of previous calculations is more helpful than providing only an oral record. By implication, instructors should supplement written worked examples with verbal explanations.

Skipping Steps

To skip steps in the solution process one must do mental calculations. Practicing problems over and over is necessary to reach the point where one can do mental calculations and thus skip steps (Roediger & Butler, 2011; Schraw & McCrudden, 2003).

As a student solves the same problem type over and over, the student slowly begins to solve the problem using different mental processes which results in the ability to skip steps (Blessing & Anderson, 1996). In the domain of algebra, when students first learn how to solve the equation $x+2=-3$ most students need to see the written process of subtracting 2 from both sides to see why the solution is $x=-5$ (Blessing & Anderson, 1996). In other words, the student needs to see the following:

$$\begin{array}{rl} x\cancel{+2} &= -3 \\ \cancel{-2} & \quad -2 \\ \hline x &= -5 \end{array}$$

After students have done similar problem types over and over they can visualize in their mind the process of subtracting two from both sides. Through repeated practice, students will begin to use different mental processes and will have the capacity to skip steps because of their increased ability to visualize the solution process (Blessing & Anderson, 1996; Helms, 2018).

By solving the same problem type again and again learners slowly become more knowledgeable and proficient with solving that particular problem type but, to skip steps, one must be knowledgeable and skilled in the problem type domain (Koedinger & Anderson, 1990). To skip algebraic steps in the solution process, students must move beyond the novice level (Koedinger & Anderson, 1990). Relative to CLT, as students solve the same problem type over and over the student slowly moves from a novice with poorly formed schema, to an expert with well-configured schema and, well-configured schema are necessary for skipping steps (Van Merriënboer & Sweller, 2005).

Charness and Campbell (1988) did a study where participants mentally squared integers between 1 and 99. The results indicate that the majority of efficiency in solving certain problem types is due to the acquisition of performing several operations at once. As a concrete example, to solve $\frac{x}{2}+1=4$ the novice would likely take two steps and use two operations (Blessing & Anderson, 1996). The first step and the first operation would be subtracting 1 from both sides of the equation to obtain $\frac{x}{2}=3$. The second step and the second operation would be multiplying each side by 2 to obtain the solution of $x=6$ (Blessing & Anderson, 1996).

The expert in algebra would use rule composition to collapse the two operations into one, allowing the expert to skip steps and immediately obtain the correct solution of $x=6$ (Blessing & Anderson, 1996). Relative to CLT, the expert can solve the equation in one step because he or she has the knowledge elements stored in well-constructed schema which enable the expert to combine simple procedures into complex ones (Van Merriënboer & Sweller, 2005). That is, the expert has chunked several knowledge elements into a single schema consisting of a set of algebraic concepts.

Worked Examples

There is strong evidence of a positive correlation between the number of steps required to solve mathematical problems and the resources needed by the WM (Ashcraft & Krause, 2007; Raghubar, Barnes & Hecht, 2010). Many problems in calculus require several steps of calculus together with several steps of algebra. The large number of steps requires a large amount of resources by the WM. But, the aim of CLT is to reduce the overall cognitive load in the WM so well-formed schemata can be constructed in LTM (Pass, Renkl & Sweller, 2004).

Relative to mathematics, worked examples is an effective approach for learning because it reduces the load in the WM while aiding in schema acquisition (Carroll, 1994; Gerjets, Scheiter & Catrambone, 2004; Stark, Mandl, Gruber & Renkl, 2002; Zhu & Simon, 1987). By effectively writing worked examples, part of which includes writing every step in the solution process, the load in the WM is reduced for learners because the learners can visually see the process needed to solve the problem. In other words, a visual [written] record of previous calculations is helpful because the written work serves as a temporary external hard drive for the WM (Cooper, 1998; Lindsay & Norman, 2013).

Cautions

First, though worked examples are effective for novice learners, for more advanced students, worked examples and including every intermediate step can become redundant (Kalyuga, et al., 2003; Van Merriënboer & Sweller, 2005). If the content in the worked examples is novel, redundancy should not be an issue. However, as the novice matures into an expert, teaching strategies must change. If instructional strategies do not change with the maturing student, learning can be inhibited. This phenomenon is known as the expertise reversal effect (Kalyuga et al., 2003; Paas et al., 2004). In the end, if instructors resist change and continue by using familiar and conventional formats for teaching throughout the semester, learning might be slow to manifest in students.

Second, exposing students to specific categories of worked examples helps students solve similar problems because the exposure and practice helps with schema construction (Gerjets et al., 2004; Ward & Sweller, 1990). However, it takes time for students to build well-constructed schema and to transfer the knowledge to new problem types (Ward & Sweller, 1990). Learning to balance the time it takes for students to build well-constructed schema with the possibility of the expertise reversal effect requires time, patience, and practice from instructors.

Instructional Design – Modeling with Worked Examples

According to CLT, when relevant information enters the WM the goal is to encode and store the information as schema in LTM for later retrieval (Paas & Ayres, 2014). When needed, information in LTM is retrieved and moved into the WM for conscious work. After the information is processed in the WM the information is stored again in LTM (Kirschner, 2002; Kirschner, Kirschner & Paas, 2009). As this process continues, the schema in LTM becomes well configured, more transparent, more complex, and contains more information elements (Van Merriënboer & Sweller, 2005).

When a complex schema is retrieved from LTM and moved to the WM the schema, which now contains several elements, is processed in the WM as one element (Van Merriënboer & Sweller, 2005). In the domain of algebra, the first time a student is asked to factor a trinomial, each operation (adding, subtracting, and multiplying) will be processed as a single information element in the WM (Paas & Ayres, 2014). Because the WM can process only a few novel elements at one time, when first exposed to factoring trinomials the load in the WM is high (Paas & Ayres, 2014). Each time another trinomial is factored, the structure of the schema in LTM becomes more finished. After factoring many trinomials over a long period of time the associated schema becomes more complete, better constructed, and contains several information elements (Ericsson & Charness, 1994; Simon & Gilmartin, 1973). In other words, the associated schema is now a complex schema containing several related concepts and is treated as a single information element when moved to the WM for processing. Through exposure and practice, factoring trinomials becomes easier over time because the schema can process more operations when moved into the WM (Roediger & Butler, 2011). Viz., during the process of factoring many trinomials, the working relationship between the WM and LTM is not only utilized, but strengthened (Paas & Ayres, 2014).

When a complex schema in LTM is moved into the WM, the associated information element in the WM is also complex (Chi et al., 1981). Complex information elements can suggest a high germane load but, a well-constructed complex information element actually reduces the overall load in the WM because the related knowledge elements act as a single element when processed in the WM (Van Merriënboer & Sweller, 2005). A high germane load can potentially reduce the overall cognitive load in the WM. Relative to algebra, the germane load in factoring $x^2 - 13x - 30$ is relatively high due to the number of operations required (adding, subtracting, and multiplying with positive and negative whole numbers) (Paas, Renkl & Sweller, 2003). Because the expert can mentally add, subtract, and multiply quickly and efficiently all three operations act as a single knowledge element in the WM. Accordingly, the overall load in the WM is low for the expert (Blessing & Anderson, 1996; Van Merriënboer & Sweller, 2005).

On the contrary, if skill acquisition has not been achieved the associated schema will be unstructured (Van Gog, Ericsson, Rikers & Paas, 2005). If schema in LTM have poor form, when moved into the WM, the WM becomes easily overloaded and learning slows to a crawl because every element must be processed separately (Van Merriënboer & Sweller, 2005). For example, because the novice learner is unable to quickly and efficiently add, subtract and multiply integers, when factoring $x^2 - 13x - 30$ each operation must be done separately in the WM (Blessing & Anderson, 1996). In other words, the information elements in the WM are handled as three separate knowledge elements (adding, subtracting, and multiplying) making the overall load in the WM high.

Example Based Instruction

Because the aim of CLT is to find effective strategies for instruction, the aim of instructional design through CLT is to find teaching strategies for handling the load in the WM (Van Gog et al., 2005). One way to help manage the load in the WM is to create an environment where schema construction is optimized. Empirical evidence suggests that example based instruction is an effective approach for building schema in the early stages of schema acquisition, when the learner is a novice (Gerjets et al., 2004; Paas, Renkl & Sweller, 2004).

Novice learners need to be exposed to and work with many examples to reach the point of well-constructed schema (Cummings, 1992; Van Merriënboer & Sweller, 2005). To help students construct well-configured and complex schemas, instructors must first introduce the topic by presenting the necessary contextual knowledge, concepts, principles, guidelines, etc. (Cooper, 1998). Then, the instructor should do worked examples to demonstrate how the contextual knowledge, concepts, principles, etc. are applied. Last, students must follow up by practicing several similar problems for homework (Cooper, 1998; Ericsson & Charness, 1994; Simon & Gilmartin, 1973).

The degree to which one can construct well-formed schemata that hold relevant and specific problem solving information defines the individual's degree of expertise (Chi, Glaser & Rees, 1981; De Groot & de Groot, 1978; Sweller, Mawer & Ward, 1983). Relative to algebra, once the novice learner acquires well-constructed algebraic schemata the novice is no longer a novice and the learner moves closer to the expert level. As an expert, the algebraic schemata in LTM now contain several elements and can be processed more easily when moved into the WM (Van Merriënboer & Sweller, 2005). This idea of progressive hierarchical learning is reinforced by Bloom's Revised Taxonomy and the Structure of the Knowledge Dimension (Krathwohl, 2002).

When confronted with an equation, an expert in algebra can recognize the category type and procedure needed to solve the equation because the expert has declarative, conceptual, and procedural knowledge (Hiebert, 2013; Hjørland & Pedersen, 2005; Krathwohl, 2002; Ritter et al., 2007). The ability to identify category and process types is a result of well-constructed schema (Hiebert, 2013; Hjørland & Pedersen, 2005). For example, if an equation contains a trinomial quadratic, the expert can quickly determine it is in fact a trinomial quadratic (categorical classification). The expert can also determine if factoring or the quadratic formula (procedural knowledge) is most appropriate (Hiebert, 2013; Hjørland & Pedersen, 2005; Krathwohl, 2002).

Adding Verbal Explanations

According to Cohen (2014), when learning a new language (and mathematics is a language) beginners prefer that every word be interpreted and defined. According to Parameswaran (2010), through example based instruction, examples play a major role in comprehending and understanding the language of mathematics. Understanding the language fosters understanding of mathematics itself (Parameswaran, 2010). It is therefore important for instructors to carefully define mathematical terminology in a manner that students can comprehend and understand. If students do not understand the language of mathematics (i.e. algebra and calculus) they will encounter difficulty in comprehending exactly what a question is asking while doing homework and exams.

Furthermore, schema development, as acquired through example based instruction, can be enhanced when coupled with verbal explanations (Van Gog et al., 2005). Coupling written worked examples with verbal explanations is typical in the traditional calculus classroom. By coupling written examples with verbal explanations instructors can reinforce and make clear to students how each step is done and why (Kirschner, Sweller & Clark, 2006). Understanding the how and why behind each step helps students understand both concepts and procedures which leads to more meaningful learning (Krathwohl, 2002; Van Gog et al., 2005).

Guided Instruction

Including verbal explanations with written worked examples results in guided instruction when the instructor serves as a model or coach (Kirschner, Sweller & Clark, 2006; Mayer, 2011). Guided instruction increases the overall effectiveness of teaching and learning (Kirschner, Sweller & Clark, 2006). According to sociocultural theory, guided instruction should also include frequent interaction between instructors and students (Scott & Palincsar, 2009). For instance, while solving an un-factorable trinomial quadratic equation, instructors could ask students "Why does the next step require the quadratic formula?" The question actively guides and engages students. Sweller and Sweller (2006) suggest how obtaining knowledge from others, such as the instructor, and then incorporating that knowledge into one's own knowledge base is an effective way of obtaining and making sense of information.

Organizing Classroom Instruction

A well-organized, written classroom presentation fosters understanding (Van Merriënboer & Sweller, 2005). Even though this might be "common sense" many mathematics instructor's presentations are not written in a well-organized manner. Nevertheless, if instructors are cognizant as they teach, a well-written and organized presentation can be easily attained. To help with motivation, some of the reasons why a well-organized presentation promotes learning will be discussed.

Visually isolating, or chunking, each subtask (subgoal) of a worked problem increases understanding (Catrambone, 1995; Cooper 1998). While visually isolating each written subtask, instructors need to write the intermediate steps within each subtask. The written format of each subtask should also be well organized to help students construct well-formed schema (Van Merriënboer & Sweller, 2005). Figure 1 shows a well-organized teaching example for a maximum minimum problem by chunking each subtask and including almost every step, which facilitates understanding by aiding the construction of well-formed schema (Catrambone, 1995; Van Merriënboer & Sweller, 2005).

In Figure 1, almost every intermediate step is included but, some parts such as subtracting 4 – 16, and solving $x - 2 = 0$ and $x + 2 = 0$, the intermediate steps were not explicitly written and should therefore be explained verbally during the solution process (Kirschner, Sweller & Clark, 2006; Mayer, 2011). The reason these steps were skipped is because of the point in time where maximum minimum problems are covered in a typical first semester calculus course; towards the middle of the semester. Since maximum minimum problems are typically covered towards the middle of first semester calculus, students are likely to be changing from a novice level of understanding algebra to an intermediate level. As students' abilities change, instruction should change with it (Kalyuga, Ayres, Chandler & Sweller, 2003; Van Merriënboer & Sweller, 2005). As students slowly become more proficient in algebra, more steps should gradually be left out so the students are not "bored" and the expertise reversal effect is avoided (Kalyuga, Ayres, Chandler & Sweller, 2003; Van Merriënboer & Sweller, 2005). However, including a verbal

Example: Find the critical numbers of $f(x) = \dfrac{x-1}{x^2-4}$.

Solution: Begin by finding the first derivative (use the Quotient Rule): $f'(x) = \dfrac{(x^2-4)(1)-(x-1)(2x)}{(x^2-4)^2}$

Next, simplify the numerator: $f'(x) = \dfrac{(x^2-4)(1)-(x-1)(2x)}{(x^2-4)^2}$

$$= \frac{x^2-4-(2x^2-2x)}{(x^2-4)^2}$$

$$= \frac{x^2-4-2x^2+2x}{(x^2-4)^2}$$

$$= \frac{-x^2+2x-4}{(x^2-4)^2}$$

By definition, to find the critical numbers, set the numerator and the denominator equal to zero and solve:

$$-x^2+2x-4=0$$

Multiply both sides by −1:

$$x^2-2x+4=0$$

Use the quadratic formula:

$$x = \frac{2 \pm \sqrt{4-16}}{2}$$

$$= \frac{2 \pm \sqrt{-12}}{2}$$

$$= \frac{2 \pm \sqrt{-4 \cdot 3}}{2}$$

$$= \frac{2 \pm 2i\sqrt{3}}{2}$$

$$(x^2-4)^2 = 0$$

Apply the square root property:

$$x^2-4=0$$

Factor the left side:

$$(x-2)(x+2)=0$$

Set each factor equal to 0 and solve:

$$x-2=0 \qquad x+2=0$$

$$x=2 \qquad x=-2$$

Because $\dfrac{2 \pm 2i\sqrt{3}}{2}$ contains complex numbers, neither solution is a critical number. Therefore, the only critical numbers are $x = 2$ and $x = -2$.

Figure 1: Example of Finding Critical Numbers. Each subtask in the worked out solution is chunked together.

explanation of every skipped step is highly recommended throughout the entire semester (Kirschner, Sweller & Clark, 2006).

In Figure 1, when simplifying the numerator of $f'(x)$ it could be argued that $2x$ and the negative sign could have been distributed in a single step. If the simplification was done this way, while writing the step-by-step procedure, a verbal explanation should be included (Kirschner, Sweller & Clark, 2006).

Discussion and Implications

Instructional Design

Because students enter first semester calculus with weak algebra skills, the question arises, “What can be done in the classroom to improve algebra skills while learning the calculus?” This question can be addressed by considering Cognitive Load Theory (CLT).

Due to students’ weak algebra skills, instructional practices should be modified to maximize learning in calculus. Specifically, skipping algebraic steps should be avoided, especially towards the beginning of the semester when students are weak in algebra because to skip steps one must do mental calculations. To do mental calculations, students must be proficient in algebra but, towards the beginning of calculus students are not proficient (Roediger & Butler, 2011; Schraw & McCrudden, 2003). It takes time and repeated practice to become proficient. Therefore, patience is required on behalf of instructors.

In addition to avoiding skipping steps, every intermediate step should be written on the “chalk” board and explained verbally when teaching (Blessing & Anderson, 1996). By writing every step and verbally explaining the procedure when solving worked examples during class, students will learn procedural processes. Knowing procedures is essential for novice learners to become experts (Hiebert, 2013; Hjørland & Pedersen, 2005; Krathwohl, 2002). However, procedural knowledge is only part of being an expert. Obtaining conceptual understanding is also of the essence. To aid in understanding, instructors must define the mathematical terminology being used.

In Martin’s (2000) study, students’ strongest performance in calculus was closely linked to procedures. On the contrary, the weakest performance was linked to problems requiring conceptual understanding. Martin found calculus students to be strongest at executing algorithms and procedures but, students were inept when it came to applying algorithms and procedures to underlying concepts. Martin also discussed how calculus students lack the understanding of the fundamental concepts of a variable. Students were found to use constants when a variable should have been used and vice versa. Thompson (1994) discovered how students are inclined to manipulate variables without thinking about the variables’ significance and what the variables represent. Again, it is vital that instructors thoughtfully define the mathematical terminology.

Empirical evidence shows a causal relationship between conceptual knowledge and procedural knowledge relative to learning and understanding mathematics (Rittle-Johnson & Alibali, 1999). The findings by Rittle-Johnson and Alibali (1999) suggest that conceptual understanding has a bigger effect on procedural understanding than procedural understanding has on conceptual. Therefore, when teaching algebraic manipulations, conceptual understanding must be integrated into the procedures. For example, to solve the equation $2x = 8$, the procedure is to divide both sides by 2. The concept however, involves the inverse operation. Explaining that the inverse (or opposite) of multiplying is dividing can help students understand why dividing both sides by 2 is necessary. Likewise, the procedure for solving $\frac{x}{2} = 8$ is to multiply both sides by 2 because the inverse of dividing is multiplying.

Instructors should also include verbal explanations about why (conceptual understanding) an algebraic procedure is used to solve a specific problem type (Kirschner, Sweller & Clark, 2006). Explaining “why” helps students’ understanding and assists in building well-configured schemas which aid in students learning how to classify problem types. (Hjørland & Pedersen, 2005). When students possess the knowledge to classify equations, when a classification is made, the probability of being correct will be elevated (Hjørland & Pedersen, 2005). If students are unable to classify equations, when encountered with an equation students will be less likely to solve it correctly because proper classification is needed to correctly solve equations. In other words, the ability to properly classify equations is a prerequisite to applying the necessary algebraic procedures (Krathwohl, 2002).

In addition to explaining “why”, when solving worked examples during class, instructors should chunk each subtask on the “chalk” board to facilitate student understanding (Catrambone, 1995; Cooper 1998). Each chunk should also be written in an organized format. Chunking each subtask in a well-organized manner helps students construct well-configured schemas (Van Merriënboer & Sweller, 2005). Well-configured schemas allows students to recognize problem types and retrieve the appropriate information in LTM to solve the problems (Kalyuga & Sweller, 2004).

Students' algebra skills gradually increase throughout the semester. To reduce extraneous cognitive load, instructional strategies should be modified to account for the gradual increase in algebra proficiency. As the semester progresses, instructors should gradually skip steps but, skipping steps should be completely avoided during the beginning of the semester, when students are still weak in algebra. As instructors begin to skip steps the instructors should verbalize the skipped part. For example, after applying the square root property to $(x-1)^2 = 9$, instructors would write $x-1 = \pm 3$ and verbally say "After applying the square root property the result is…" Instructors might add "Do not forget the plus and minus."

A main reason for skipping steps (when students are sufficiently prepared) is because skipping steps requires students to solve equations in their mind. By solving equations in ones' mind, students' strategies for solving equations shifts and students begin to see how solving equations in fewer steps is actually easier (Blessing & Anderson, 1996). By skipping steps students also begin to utilize shortcuts and, their overall performance increases (Blessing & Anderson, 1996). Instructors must realize however, it takes time for students to reach the point of effectively doing mental calculations. The ability to skip steps only occurs after students have applied the "rules" to several cases over a long period of time (Blessing & Anderson, 1996).

References

Adams, J., & Hitch, G. (1997). Working memory and children's mental addition. *Journal of experimental child psychology*, *67*(1), 21-38.

Ambrose, S., Bridges, M., DiPietro, M., Lovett, M., & Norman, M. (2010). *How learning works: Seven research-based principles for smart teaching*. John Wiley & Sons.

Anderson, J. (2013). *The architecture of cognition*. Psychology Press.

Ashcraft, M., & Krause, J. (2007). Working memory, math performance, and math anxiety. *Psychonomic bulletin & review*, *14*(2), 243-248.

Avila, C. (2013). *Calculus instructor's assumptions of their student's prior knowledge of functions: A multiple-case study* (Doctoral dissertation). University of Central Florida Orlando, Florida.

Bailey, T., Jeong, D. W., & Cho, S. W. (2010). Referral, enrollment, and completion in developmental education sequences in community colleges. *Economics of Education Review*, *29*(2), 255-270.

Blessing, S., & Anderson, J. (1996). How people learn to skip steps. *Journal of experimental psychology: learning, memory, and cognition*, *22*(3), 576.

Budny, D., Bjedov, G. & LeBold, W. (1998). Assessment of the impact of freshman engineering courses. *Journal of Engineering Education*, *87*(4), 405-411.

Bull, R., & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology*, 19(3), 273-293.

Carrell, S. E., & West, J. E. (2008). *Does professor quality matter? Evidence from random assignment of students to professors* (No. w14081). National Bureau of Economic Research.

Carroll, W. (1994). Using worked examples as an instructional support in the algebra classroom. *Journal of Educational Psychology*, *86*(3), 360.

Catrambone, R. (1995). Aiding subgoal learning: Effects on transfer. *Journal of educational psychology*, *87*(1), 5.

Charness, N., & Campbell, J. (1988). Acquiring skill at mental calculation in adulthood: A task decomposition. *Journal of Experimental Psychology: General*, *117*(2), 115.

Chi, M., Glaser, R., & Rees, E. (1981). *Expertise in problem solving* (No. TR-5). Pittsburgh Univ. PA Learning Research and Development Center.

Cohen, A. D. (2014). *Strategies in learning and using a second language*. Routledge.

Cooper, G. (1998). Research into cognitive load theory and instructional design at UNSW. *Retrieved December 1*, 2015.

Daempfle, P. A. (2003). An analysis of the high attrition rates among first year college science, math, and engineering majors. *Journal of College Student Retention: Research, Theory & Practice*, *5*(1), 37-52.

De Groot, A., & De Groot, A. (1978). *Thought and choice in chess* (Vol. 4). Walter de Gruyter.

Ericsson, K., & Charness, N. (1994). Expert performance: Its structure and acquisition. *American psychologist*, *49*(8), 725.

Gerjets, P., Scheiter, K., & Catrambone, R. (2004). Designing instructional examples to reduce intrinsic cognitive load: Molar versus modular presentation of solution procedures. *Instructional Science*, *32*(1-2), 33-58.

Gobet, F. (2005). Chunking models of expertise: Implications for education. *Applied Cognitive Psychology*, *19*(2), 183-204.

Heid, M. K., & Edwards, M. T. (2001). Computer algebra systems: revolution or retrofit for today's mathematics classrooms?. *Theory into practice*, *40*(2), 128-136.

Helms, J. (2018). Algebra Proficiency in Calculus: A Look at English Language Learners. Proceedings from HUIC '18: *Arts, Humanities, Social Sciences & Education*, Honolulu, HI. ISSN #2162-9188

Herriott, S. R., & Dunbar, S. R. (2009). Who takes college algebra? *Primus*, *19*(1), 74-87.

Hiebert, J. (2013). *Conceptual and procedural knowledge: The case of mathematics*. Routledge.

Hitch, G. (1978). The role of short-term working memory in mental arithmetic. *Cognitive Psychology*, *10*(3), 302-323.

Hjørland, B., & Nissen Pedersen, K. (2005). A substantive theory of classification for information retrieval. *Journal of documentation*, *61*(5), 582-597.

Jourdan, N., Cretchley, P., & Passmore, T. (2007). Secondary-tertiary transition: What mathematics skills can and should we expect this decade?. In *Proceedings of the 30th Annual Conference of the Mathematics Education Research Group of Australasia (MERGA 30)* (Vol. 2, pp. 463-472). Mathematics Education Research Group of Australasia (MERGA).

Kalyuga, S., Ayres, P., Chandler, P., & Sweller, J. (2003). The expertise reversal effect. *Educational psychologist*, *38*(1), 23-31.

Kalyuga, S., & Sweller, J. (2004). Measuring knowledge to optimize cognitive load factors during instruction. *Journal of educational psychology*, *96*(3), 558.

Kirschner, P. (2002). Cognitive load theory: Implications of cognitive load theory on the design of learning. *Learning and instruction*, *12*(1), 1-10.

Kirschner, P., Kirschner, F., & Paas, F. (2009). Cognitive load theory. *Retrieved March 23*, 2015 from education.com

Kirschner, P., Sweller, J., & Clark, R. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational psychologist*, *41*(2), 75-86.

Koedinger, K., & Anderson, J. (1990). Abstract planning and perceptual chunks: Elements of expertise in geometry. *Cognitive Science*, *14*(4), 511-550.

Krathwohl, D. (2002). A revision of Bloom's taxonomy: An overview. *Theory into practice*, *41*(4), 212-218.

Lindsay, P., & Norman, D. (2013). *Human information processing: An introduction to psychology*. Academic Press.

Martin, T. (2000). Calculus students' ability to solve geometrical related rates problems. *Mathematics Education Research Journal*, *12*(2), 74-91.

Mayer, R. (2011). *Applying the science of learning*. Boston: Pearson/Allyn & Bacon.

Orton, A. (1983). Students' understanding of differentiation. *Educational Studies in Mathematics*, *14*(3), 235-250.

Paas, F., & Ayres, P. (2014). Cognitive load theory: A broader view on the role of memory in learning and education. *Educational Psychology Review*, *26*(2), 191-195.

Paas, F., Renkl, A., & Sweller, J. (2003). Cognitive load theory and instructional design: Recent developments. *Educational psychologist*, *38*(1), 1-4.

Paas, F., Renkl, A., & Sweller, J. (2004). Cognitive load theory: Instructional implications of the interaction between information structures and cognitive architecture. *Instructional science*, *32*(1), 1-8.

Paas, F., Van Gog, T., & Sweller, J. (2010). Cognitive load theory: New conceptualizations, specifications, and integrated research perspectives. *Educational Psychology Review*, *22*(2), 115-121.

Paas, F., & Van Merriënboer, J. (1994). Variability of worked examples and transfer of geometrical problem-solving skills: A cognitive-load approach. *Journal of educational psychology*, *86*(1), 122.

Parameswaran, R. (2010). Expert Mathematics' Approach to Understanding Definitions. *Mathematics Educator*, *20*(1), 43-51.

Pollock, E., Chandler, P., & Sweller, J. (2002). Assimilating complex information. *Learning and instruction*, *12*(1), 61-86.

Raghubar, K., Barnes, M., & Hecht, S. (2010). Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches. *Learning and Individual Differences*, *20*(2), 110-122.

Ritter, S., Anderson, J. R., Koedinger, K. R., & Corbett, A. (2007). Cognitive Tutor: Applied research in mathematics education. *Psychonomic bulletin & review*, *14*(2), 249-255.

Rittle-Johnson, B., & Alibali, M. W. (1999). Conceptual and procedural knowledge of mathematics: Does one lead to the other?. *Journal of educational psychology*, *91*(1), 175.

Roediger, H. L., & Butler, A. C. (2011). The critical role of retrieval practice in long-term retention. *Trends in cognitive sciences*, *15*(1), 20-27.

Salden, R., Paas, F., & Van Merriënboer, J. (2001). Selection of Learning Tasks Based on Performance and Cognitive Load Scores as a Way to Optimize the Learning Process.

Schraw, G., & McCrudden, M. (2003). Information processing theory. *Retrieved February 27*, 2015 from education.com

Scott, S., & Palincsar, A. (2009). Sociocultural theory. *Retrieved April 24*, 2015 from education.com

Seymour, E., & Hewitt, N. M. (1994). *Talking about leaving: factors contributing to high attrition rates among science, mathematics & engineering undergraduate majors: final report to the Alfred P. Sloan Foundation on an ethnographic inquiry at seven institutions*. Ethnography and Assessment Research, Bureau of Sociological Research, University of Colorado.

Simon, H., & Gilmartin, K. (1973). A simulation of memory for chess positions. *Cognitive psychology*, *5*(1), 29-46.

Stark, R., Mandl, H., Gruber, H., & Renkl, A. (2002). Conditions and effects of example elaboration. *Learning and Instruction*, *12*(1), 39-60.

Stewart, J. (2015). *Calculus: Early transcendentals* (8th ed.). Brooks/Cole Cengage Learning.

Sweller, J., Mawer, R. F., & Ward, M. R. (1983). Development of expertise in mathematical problem solving. *Journal of Experimental Psychology: General*,*112*(4), 639.

Sweller, J., & Sweller, S. (2006). Natural information processing systems. *Evolutionary Psychology*, *4*(2), 434-58.

Tall, D. (1993). Students' difficulties in calculus. In *proceedings of working group* (Vol. 3, pp. 13-28).

Tan, S. (2011). *Single variable calculus: Early transcendentals*. Belmont, CA: Brooks/Cole Cengage Learning.

Thompson, P. W. (1994). Images of rate and operational understanding of the fundamental theorem of calculus. In *Learning Mathematics* (pp. 125-170). Springer Netherlands.

Van Gog, T., Ericsson, K., Rikers, R., & Paas, F. (2005). Instructional design for advanced learners: Establishing connections between the theoretical frameworks of cognitive load and deliberate practice. *Educational Technology Research and Development*, *53*(3), 73-81.

Van Merriënboer, J., & Sweller, J. (2005). Cognitive load theory and complex learning: Recent developments and future directions. *Educational psychology review*, *17*(2), 147-177.

Ward, M., & Sweller, J. (1990). Structuring effective worked examples. *Cognition and instruction*, *7*(1), 1-39.

Zhu, X., & Simon, H. (1987). Learning mathematics from examples and by doing. *Cognition and instruction*, *4*(3), 137-166.